

BAREM DE CORECTARE ȘI NOTARE

Clasa a V-a

1. **a) (3p)** Arătați că numărul $x = (3^{1+2+3+\dots+42} + 2 \cdot 3^{1+3+5+\dots+59}) : 29$ este cub perfect.
- b) (4p)** Arătați că numărul x poate fi scris ca o sumă de patru pătrate perfecte nenule.

Prof. Sascău Gabriela

Soluție:

- a)** $1 + 2 + 3 + \dots + 42 = 42 \cdot 43 : 2 = 903$
 $1 + 3 + 5 + \dots + 59 = 1 + 2 + 3 + \dots + 59 - (2 + 4 + \dots + 58) = 59 \cdot 60 : 2 - 2 \cdot 29 \cdot 30 : 2 = 30^2 = 900$
 $x = (3^{903} + 2 \cdot 3^{900}) : 29 = 3^{900}(27 + 2) : 29 = 3^{900} = (3^{300})^3$
- b)** $x = 3^{900} = 3^{896} \cdot 3^4 = 3^{896} \cdot 81$. Dar 81 nu se poate scrie ca sumă a patru pătrate perfecte nenule.
 $x = 3^{896} \cdot 3^4 = 3^{896} \cdot 81 = 3^{896} \cdot (4 + 16 + 25 + 36) = (3^{448} \cdot 2)^2 + (3^{448} \cdot 4)^2 + (3^{448} \cdot 5)^2 + (3^{448} \cdot 6)^2$

Barem

a) $3^{1+2+3+\dots+42} = 3^{903}; 3^{1+3+5+\dots+59} = 3^{900}$	1p
$x = 3^{900}(27 + 2) : 29 = 3^{900}$	1p
$x = (3^{300})^3$	1p
b) $x = 3^{900} = 3^{896} \cdot 3^4 = 3^{896} \cdot 81 = 3^{896} \cdot (4 + 16 + 25 + 36)$	2p
$x = 3^{896} \cdot 2^2 + 3^{896} \cdot 4^2 + 3^{896} \cdot 5^2 + 3^{896} \cdot 6^2 = (3^{448} \cdot 2)^2 + (3^{448} \cdot 4)^2 + (3^{448} \cdot 5)^2 + (3^{448} \cdot 6)^2$	2p

2. Se consideră mulțimile $A = \{5n + 4; 5n + 5; 5n + 9 / n \in \mathbb{N}\}$ și $B = \{m^2 + 2017 / m \in \mathbb{N}\}$.

- a) (2p)** Stabiliți dacă $2017 \in A$.
- b) (5p)** Calculați $A \cap B$.

Prof. Schroder Laura

Soluție:

- a)** $2017 = 5 \cdot 403 + 2$ deci $2017 \notin A$.
- b)** $u(m^2) \in \{0, 1, 4, 5, 6, 9\}$ prin urmare $u(m^2 + 2017) \in \{1, 2, 3, 6, 7, 8\}$.
 $u(5 \cdot n) \in \{0, 5\}$ prin urmare $u(A) \in \{0, 4, 5, 9\}$, deci $A \cap B = \emptyset$.

Barem

a)	$2017 = 5 \cdot 403 + 2$ $2017 \notin A$	1p
b)	$u(m^2) \in \{0, 1, 4, 5, 6, 9\}$	1p

$u(m^2 + 2017) \in \{1, 2, 3, 6, 7, 8\}$	1p
$u(5 \cdot n) \in \{0, 5\}$	1p
$u(A) \in \{0, 4, 5, 9\}$	1p
$A \cap B = \emptyset$	1p

3. a) (3p) Arătați că $\overbrace{abab\dots ab}^{2016 \text{ litere}} + \overbrace{baba\dots ba}^{2016 \text{ litere}} = \overbrace{aa\dots a}^{\text{de } 2016 \text{ ori}} + \overbrace{bb\dots b}^{\text{de } 2016 \text{ ori}}$.

b) (4p) Demonstrați că $37 / \left(\overbrace{abab\dots ab}^{2016 \text{ litere}} + \overbrace{baba\dots ba}^{2016 \text{ litere}} \right)$.

Prof. Ciobîcă Constantin

Soluție:

a)

$$\begin{aligned} \overbrace{abab\dots ab}^{2016 \text{ litere}} + \overbrace{baba\dots ba}^{2016 \text{ litere}} &= a \cdot \underbrace{100\dots 0}_{\text{de } 2015 \text{ ori}} + b \cdot \underbrace{100\dots 0}_{\text{de } 2014 \text{ ori}} + a \cdot \underbrace{100\dots 0}_{\text{de } 2013 \text{ ori}} + b \cdot \underbrace{100\dots 0}_{\text{de } 2012 \text{ ori}} + \dots + a \cdot 10 + b + \\ &+ b \cdot \underbrace{100\dots 0}_{\text{de } 2015 \text{ ori}} + a \cdot \underbrace{100\dots 0}_{\text{de } 2014 \text{ ori}} + b \cdot \underbrace{100\dots 0}_{\text{de } 2013 \text{ ori}} + a \cdot \underbrace{100\dots 0}_{\text{de } 2012 \text{ ori}} + \dots + b \cdot 10 + a = \\ &= a \cdot \underbrace{100\dots 0}_{\text{de } 2015 \text{ ori}} + a \cdot \underbrace{100\dots 0}_{\text{de } 2014 \text{ ori}} + \dots + a \cdot 10 + a + b \cdot \underbrace{100\dots 0}_{\text{de } 2015 \text{ ori}} + b \cdot \underbrace{100\dots 0}_{\text{de } 2014 \text{ ori}} + \dots + b \cdot 10 + b = \\ &= \overbrace{aa\dots a}^{\text{de } 2016 \text{ ori}} + \overbrace{bb\dots b}^{\text{de } 2016 \text{ ori}}. \end{aligned}$$

b) $\overbrace{aa\dots a}^{\text{de } 2016 \text{ ori}} + \overbrace{bb\dots b}^{\text{de } 2016 \text{ ori}} = a \cdot \underbrace{11\dots 1}_{\text{de } 2016 \text{ ori}} + b \cdot \underbrace{11\dots 1}_{\text{de } 2016 \text{ ori}} = \underbrace{11\dots 1}_{\text{de } 2016 \text{ ori}} \cdot (a + b) = 111 \cdot \underbrace{1001001\dots 1001}_{2014 \text{ cifre}} \cdot (a + b) =$
 $= 37 \cdot 3 \cdot \underbrace{1001001\dots 1001}_{2014 \text{ cifre}} \cdot (a + b)$

Barem

a)	$\overbrace{abab\dots ab}^{2016 \text{ litere}} = a \cdot \underbrace{100\dots 0}_{\text{de } 2015 \text{ ori}} + b \cdot \underbrace{100\dots 0}_{\text{de } 2014 \text{ ori}} + a \cdot \underbrace{100\dots 0}_{\text{de } 2013 \text{ ori}} + b \cdot \underbrace{100\dots 0}_{\text{de } 2012 \text{ ori}} + \dots + a \cdot 10 + b$	1p
	$\overbrace{baba\dots ba}^{2016 \text{ litere}} = b \cdot \underbrace{100\dots 0}_{\text{de } 2015 \text{ ori}} + a \cdot \underbrace{100\dots 0}_{\text{de } 2014 \text{ ori}} + b \cdot \underbrace{100\dots 0}_{\text{de } 2013 \text{ ori}} + a \cdot \underbrace{100\dots 0}_{\text{de } 2012 \text{ ori}} + \dots + b \cdot 10 + a$	1p
	$\overbrace{abab\dots ab}^{2016 \text{ litere}} + \overbrace{baba\dots ba}^{2016 \text{ litere}} = a \cdot \underbrace{100\dots 0}_{\text{de } 2015 \text{ ori}} + a \cdot \underbrace{100\dots 0}_{\text{de } 2014 \text{ ori}} + \dots + a \cdot 10 + a +$ $+ b \cdot \underbrace{100\dots 0}_{\text{de } 2015 \text{ ori}} + b \cdot \underbrace{100\dots 0}_{\text{de } 2014 \text{ ori}} + \dots + b \cdot 10 + b = \overbrace{aa\dots a}^{\text{de } 2016 \text{ ori}} + \overbrace{bb\dots b}^{\text{de } 2016 \text{ ori}}$	1p
b)	$\overbrace{aa\dots a}^{\text{de } 2016 \text{ ori}} + \overbrace{bb\dots b}^{\text{de } 2016 \text{ ori}} = a \cdot \underbrace{11\dots 1}_{\text{de } 2016 \text{ ori}} + b \cdot \underbrace{11\dots 1}_{\text{de } 2016 \text{ ori}} = \underbrace{11\dots 1}_{\text{de } 2016 \text{ ori}} \cdot (a + b)$	1p
	$\underbrace{11\dots 1}_{\text{de } 2016 \text{ ori}} = 111 \cdot \underbrace{1001001\dots 1001}_{2014 \text{ cifre}} = 37 \cdot 3 \cdot \underbrace{1001001\dots 1001}_{2014 \text{ cifre}}$	2p
	Finalizare $37 / \left(\overbrace{abab\dots ab}^{2016 \text{ litere}} + \overbrace{baba\dots ba}^{2016 \text{ litere}} \right)$	1p

Notă: Orice altă soluție corectă se va puncta corespunzător.